Model Questions

Department of Physics
Government General Degree College, Chapra
Course Title- Mathematical Physics
Semester – I

1. (a) Plot the function $f(x) = x^2$ and its first derivative. 2

   (b) Find the series expansion of $\frac{1}{1-x}$. Mention its interval of convergence. 2+1

2. (a) Check whether $dw = 2xydx + x^2dy$ is an exact differential. 2

   (b) Solve the differential equation :- $(D^2 + 1)$y = cosx + $e^x sinx$ 3

3. (a) Solve the differential equation $(x+1)\frac{dy}{dx} - y = e^x\,(x+1)^2$. 3

   (b) Find the Taylor series expansion of lnx about $x = 2$. 2

4. Solve the equation –

   $y'' + 6y' + 8y = 0$, subject to the condition $y = 1, y' = 0$ at $x = 0$,

   where, $y' = \frac{dy}{dx}$ and $y'' = \frac{d^2y}{dx^2}$. 5

5. (a) Find a unit vector normal to $\vec{A} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\vec{B} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. 2

   (b) Using Stoke's law, prove that $\vec{\nabla} \times \vec{\nabla}\emptyset = 0$. 3

6. (a) The position vectors of three points *A, B* and *C* are $\vec{r_1} = \hat{\imath} + 2\hat{\jmath} + \hat{k}, \vec{r_2} = 3\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{r_3} = 2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$. Find the area of the triangle. 3

   (b) $\vec{\omega}$ is a constant vector and $\vec{r}$ is the position vector of a point. If $\vec{v} = \vec{\omega} \times \vec{r}$, then prove that $\vec{\nabla}.\vec{v} = 0$. 2

7. $If\ \vec{F} = x^2\hat{\imath} + y^2\hat{\jmath}$, then find the line integral $\int_c \vec{F}.d\vec{r}$ in the *x* - *y* plane along a line $y = x^2$ from *P(0,0)* to *Q(1,1)*. 5

8. (a) Find the eigenvalues of the matrix $\begin{pmatrix} 3 & i \\ -i & 3 \end{pmatrix}$. 2

   (b) Prove that the modulus of each characteristic root of a unitary matrix is unity. 3

9. (a) Using elementary operations, find inverse of matrix:

$$A = \begin{pmatrix} -1 & 1 & 2 \\ 0 & 2 & 1 \\ -1 & 3 & 4 \end{pmatrix} \qquad 3$$

   (b) Prove that the eigenvalue of a skew Hermitian is purely imaginary. 2

10. Solve the following 1st order differential equation: (6 x 3 = 18)

a. $(1 + x^2)dy - xy\,dx = 0$
b. $x(x - y)dy + y^2\,dx = 0$
c. $secx \frac{dy}{dx} = y + sinx$
d. $x\,logx \frac{dy}{dx} + y = 2\,logx$
e. $\frac{dy}{dx} = y\,tanx - y^2\,secx$
f. $(2xy + e^y)dx + (x^2 + xe^y)dy = 0$

11. Solve the following 2nd order differential equation: (10 x 4 = 40)

a. $[D^2 + 5D + 6][y] = e^x$
b. $[D^2 - 3D + 2][y] = e^{3x}$
c. $[D^3 + 2D^2 - D - 2][y] = e^x$
d. $[D^2 + 2D + 2][y] = \sinh x$
e. $[D^2 + 5D + 4][y] = 3 - 2x$
f. $[D^2 - 4D + 3][y] = x^3$
g. $[D^2 + 6][y] = \sin 4x$
h. $[D^2 - 5D + 6][y] = e^x\,sinx$
i. $[D^2 + 4][y] = 3x\,sinx$
j. $[D^2 + a^2][y] = \sec ax$